Domain, Range, and Period of the three main trigonometric functions:

1. $sin(x)$

   - Domain: $\mathbb{R} = (-\infty, \infty)$
   - Range: $[-1, 1]$
   - Period: 2π

2. $cos(x)$

   - Domain: $\mathbb{R}$
   - Range: $[-1, 1]$
   - Period: 2π

3. $tan(x)$

   - Domain: $\{x | x \neq \frac{\pi}{2} + k\pi, \ k = ..., -1, 0, 1, ...\} = \{x | x \neq ..., -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, ...\}$
   - Range: $\mathbb{R}$
   - Period: π

Domain, Range, and Definition of the three main inverse trigonometric functions:

1. $sin^{-1}(x)$

   - Domain: $[-1, 1]$
   - Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$
   - Definition: $\theta = sin^{-1}(x)$ means $sin(\theta) = x$ when $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

2. $cos^{-1}(x)$

   - Domain: $[-1, 1]$
   - Range: $[0, \pi]$
   - Definition: $\theta = cos^{-1}(x)$ means $cos(\theta) = x$ when $-1 \leq x \leq 1$ and $0 \leq \theta \leq \pi$

3. $tan^{-1}(x)$

   - Domain: $\mathbb{R}$
   - Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$
   - Definition: $\theta = tan^{-1}(x)$ means $tan(\theta) = x$ when $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

I. $sin(sin^{-1}(x)) = x$ when $-1 \leq x \leq 1$.

II. $sin^{-1}(sin(x)) = x$ when $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

Good I
$$sin(sin^{-1}(1/2)) = 1/2, \text{ since } -1 \leq 1/2 \leq 1$$

Bad I
$$sin(sin^{-1}(-1.8)) = \text{ undefined, since } -1.8 < -1$$

Good II: θ is in the right quadrant, and written correctly
$$sin^{-1}\left(sin\left(-\frac{\pi}{5}\right)\right) = -\frac{\pi}{5}, \text{ since } -\frac{\pi}{2} \leq -\frac{\pi}{5} \leq \frac{\pi}{2}$$

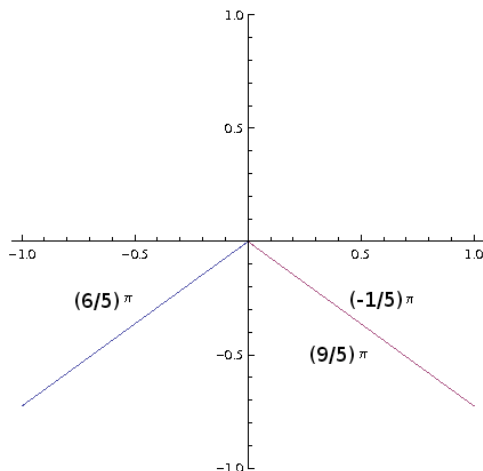**Bad II: θ is in the right quadrant, but written incorrectly**
$$sin^{-1}\left(sin\left(\frac{9\pi}{5}\right)\right) = \text{?}$$

Now $\frac{9\pi}{5}$ is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, but it *is* in the right quadrant, namely quadrant IV. To find the correct angle, simply add or subtract 2π from the angle given until you get an angle in the range of $sin^{-1}(x)$. In this case, $\frac{9\pi}{5} - 2\pi = -\frac{\pi}{5}$, so $sin^{-1}\left(sin\left(\frac{9\pi}{5}\right)\right) = -\frac{\pi}{5}$.

Worse II: θ is in the wrong quadrant
$$sin^{-1}\left(sin\left(\frac{6\pi}{5}\right)\right) = \text{?}$$

Here θ is actually in the wrong quadrant, so we need to flip it across the y axis and find the associated angle in the right quadrant. You can just look at the picture and see that $-\frac{\pi}{5}$ is the correct angle, so $sin^{-1}\left(sin\left(\frac{6\pi}{5}\right)\right) = -\frac{\pi}{5}$.

I. $cos(cos^{-1}(x)) = x$ when $-1 \leq x \leq 1$.

II. $cos^{-1}(cos(x)) = x$ when $0 \leq x \leq \pi$.

Good I
$$cos(cos^{-1}(-1/3)) = -1/3, \text{ since } -1 \leq -1/3 \leq 1$$

Bad I
$$cos(cos^{-1}(\sqrt{3})) = \text{ undefined, since } 1 < \sqrt{3}$$

Good II: θ is in the right quadrant, and written correctly
$$cos^{-1}(cos\left(\frac{4\pi}{5}\right)) = \frac{4\pi}{5}, \text{ since } 0 \leq \frac{4\pi}{5} \leq \pi$$

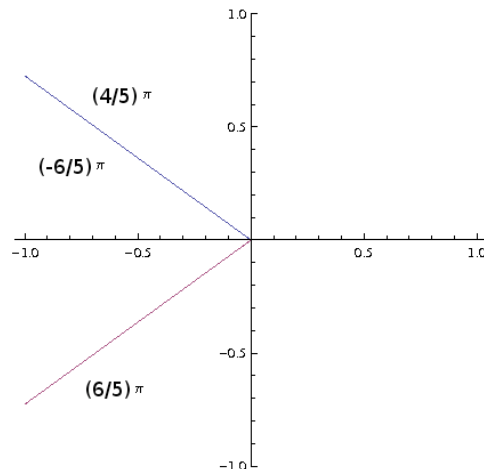**Bad II: θ is in the right quadrant, but written incorrectly**
$$cos^{-1}(cos\left(-\frac{6\pi}{5}\right)) = ?$$

Now $-\frac{6\pi}{5}$ is not between 0 and π, but it *is* in the right quadrant, namely quadrant II. To find the correct angle, simply add or subtract 2π from the angle given until you get an angle in the range of $cos^{-1}(x)$. In this case, $-\frac{6\pi}{5} + 2\pi = \frac{4\pi}{5}$, so $cos^{-1}(cos\left(-\frac{6\pi}{5}\right)) = \frac{4\pi}{5}$.

Worse II: θ is in the wrong quadrant
$$cos^{-1}(cos\left(\frac{6\pi}{5}\right)) = ?$$

Here θ is actually in the wrong quadrant, so we need to flip it across the x axis and find the associated angle in the right quadrant. You can just look at the picture and see that $\frac{4\pi}{5}$ is the correct angle, so $cos^{-1}(cos\left(\frac{6\pi}{5}\right)) = \frac{4\pi}{5}$.

I. $tan(tan^{-1}(x)) = x$ when $-\infty < x < \infty$.

II. $tan^{-1}(tan(x)) = x$ when $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Good I
$$tan(tan^{-1}(-1000)) = -1000, \text{ since } -\infty < -1000 < \infty$$

Bad I

THERE IS NO BAD I FOR INVERSE TANGENT. Case I always works!

NOTE: Now there are some serious discrepancies between Sin, Cos, and Tan. The way to think of this is that even if θ is not in the range of $tan^{-1}(x)$, it is *always* in the right quadrant. So there is only Good II and Bad II, no Worse II. That means the only thing that can go wrong is that the angle was not written correctly.

Good II: θ is written correctly

$$tan^{-1}\left(tan\left(-\frac{\pi}{5}\right)\right) = -\frac{\pi}{5}, \text{ since } -\frac{\pi}{2} < -\frac{\pi}{5} < \frac{\pi}{2}$$

Bad II: θ is written incorrectly

$$tan^{-1}\left(tan\left(\frac{6\pi}{5}\right)\right) = ?$$

Now $\frac{6\pi}{5}$ is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, so just like with the Bad II for Sin and Cos, I add or subtract the period until I get an angle that *is* in the range of $tan^{-1}(x)$. For Sin and Cos, I add or subtract 2π because that is their period. For Tan, I add or subtract π, the period of $tan(x)$. Here $\frac{6\pi}{5} - \pi = \frac{\pi}{5}$, so $tan^{-1}\left(tan\left(\frac{6\pi}{5}\right)\right) = \frac{\pi}{5}$.

Worse II: θ is in the wrong quadrant

THERE IS NO WORSE II FOR INVERSE TANGENT. Only Good II and Bad II.